Roll No. | | | | | | | | | | |

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E. /B.Tech (Full Time) - END SEMESTER EXAMINATIONS, NOV / DEC 2024

ARTIFICIAL INTELLIGENCE AND DATA SCIENCE
SEMESTER - 3
MA23C04 & DISCRETE MATHEMATICS
(Regulation 2023)

Time: 3hrs Max.Marks: 100

CO1	Understand the validity of the logical arguments, mathematical proofs and correctness of the algorithm.
CO2	Apply Combinatorial counting techniques in solving combinatorial related problems.
CO3	Use graph models and their connectivity, traversability in solving real world problems
CO4	Understand the significance of algebraic structural ideas used in coding theory and cryptography.
CO5	Apply Boolean laws and Boolean functions in combinatorial circuit designs.

BL – Bloom's Taxonomy Levels
(L1-Remembering, L2-Understanding, L3-Applying, L4-Analysing, L5-Evaluating, L6-Creating)

PART- A (10x2=20Marks)

(Answer all Questions)

Q.No.	Questions	Marks	CO	BL
1	Express the statement "$for\ every\ 'x'\ there\ exist\ a\ 'y'\ such\ that\ x^2 + y^2 \leq 100$" in symbolic form.	2	1	L2
2	Prove that $\sqrt{2}$ is irrational by giving a proof using contradiction.	2	1	L4
3	If we select 10 points in the interior of an equilateral triangle of side 1 unit, show that there must be at least two points whose distance apart is less than $\frac{1}{3}$.	2	2	L3
4	How many bit strings of length 12 contain exactly four 1s?	2	2	L2
5	How many vertices does a regular graph of degree four with ten edges have?	2	3	L1
6	For what condition the complete bipartite graph and complete graph are Hamiltonian graph?	2	3	L5
7	Show that every cyclic group is abelian.	2	4	L6
8	Define field? Give one example for finite field and infinite field.	2	4	L1
9	If a poset has a least element, then prove it is unique.	2	5	L4
10	Determine whether D_8 a Boolean algebra or not? Justify your answer.	2	5	L6

PART- B(5x 13=65Marks)

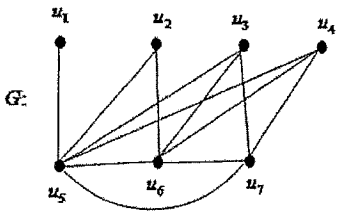
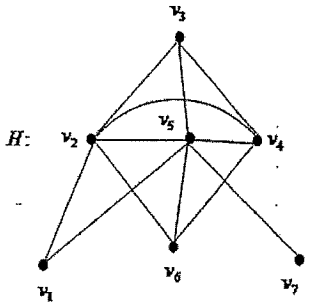
(Restrict to a maximum of 2 subdivisions)

Q.No.	Questions	Marks	CO	BL
11 (a)	(i) Obtain the PDNF and PCNF of $(\sim P \vee \sim Q) \rightarrow (P \leftrightarrow \sim Q)$.	8	1	L2
	(ii) Show that $J \wedge S$ follows logically from the premises $P \rightarrow Q$, $Q \rightarrow \sim R, R$ and $P \vee (J \wedge S)$.	5	1	L3
	OR			
11 (b)	(i) Show that the conclusion $(\forall x)(P(x) \rightarrow \sim Q(x)$ follows from the premises $(\exists x)(P(x) \wedge Q(x)) \rightarrow (\forall y)(R(y) \rightarrow S(y))$ and $(\exists y)(R(y) \wedge \sim S(y))$.	8	1	L3
	(ii) Show that "**it rained**" is a conclusion obtained from the statements. "If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, then the trophy will be awarded" and "the trophy was not awarded"	5	1	L2
12 (a)	(i)How many solutions does the equation $x_1 + x_2 + x_3 = 11$ have, where $0 \leq x_1 \leq 3$, $0 \leq x_2 \leq 4$, $0 \leq x_3 \leq 6$? Use principle of inclusion and exclusion.	8	2	L2
	(ii) Write the recurrence relation for Fibonacci number and hence solve it.	5	2	L3
	OR			
12 (b)	Solve the recurrence relative $a_n - 6a_{n-1} + 8a_{n-2} = 3^n$ where $a_0 = 3, a_1 = 7$, by using generating function.	13	2	L5
13 (a)	(i) Prove that a connected graph G is Bipartite if and only if all its cycles are of even length.	8	3	L1
	(ii) Define self complementary graph. Find the values of n so that the graph of order n is self complementary graph. Hence for what value on "n" cycle graph C_n is self complementary.	5	3	L2
	OR			
13 (b)	i) State and prove Dirac theorem.	8	3	L1
	(ii) For what value of "n" do these graphs have an Euler circuits? $(a)K_n$ $(b)C_n$ $(c)W_n$ $(d)Q_n(n-regular\ graph)$	5	3	L6
14 (a)	(i) Show that U_n is the set of n^{th} root of unity is a cyclic group under multiplication.	8	4	L1
	(ii) Prove that in a group the left and right cancellation law is true. If $a * b = a * c$ then $b = c$ and if $b * a = c * a$ then $b = c$.	5	4	L2

	OR			
14 (b)	State and Prove Cayley's Representation theorem	13	4	L1
15 (a)	(i) Prove that every chain is distributive lattice.	8	5	L1
	(ii) State that cancellation laws are valid in distributive lattice.	5	5	L3
	OR			
15 (b)	(i) State and Prove that Demorgan's law hold good for a complemented distributive lattice.	8	5	L1
	(ii) Find the values of (a) $x \oplus 0$ (b) $x \oplus 1$ (c) $x \oplus x$ (d) $x \oplus \bar{x}$.	5	5	L3

PART- C(1x 15=15Marks)

(Q.No.16 is compulsory)

Q.No.	Questions	Marks	CO	BL
16.(i)	When do we say two simple graphs are isomorphic? Check whether the following two graphs are isomorphic or not. Justify your answer.  	8	3	L5
16.(ii)	Applying Mathematical induction prove that $x^n - y^n$ has $x - y$ as a factor.	7	2	L6